1.4 Measuring Progress

Finding Fractional Parts

Here are two claims about the fundraising goals from Problem 1.1.

Ben:

For every $60 the sixth graders plan to raise, the seventh graders plan to raise $90.

Kimi:

When the sixth graders meet their goal, they will have raised $\frac{2}{3}$ of the seventh-grade goal.

Ben and Kimi are each comparing one sixth-grade goal to one seventh-grade goal. Ben uses ratios to make comparisons and Kimi uses fractions to make comparisons.

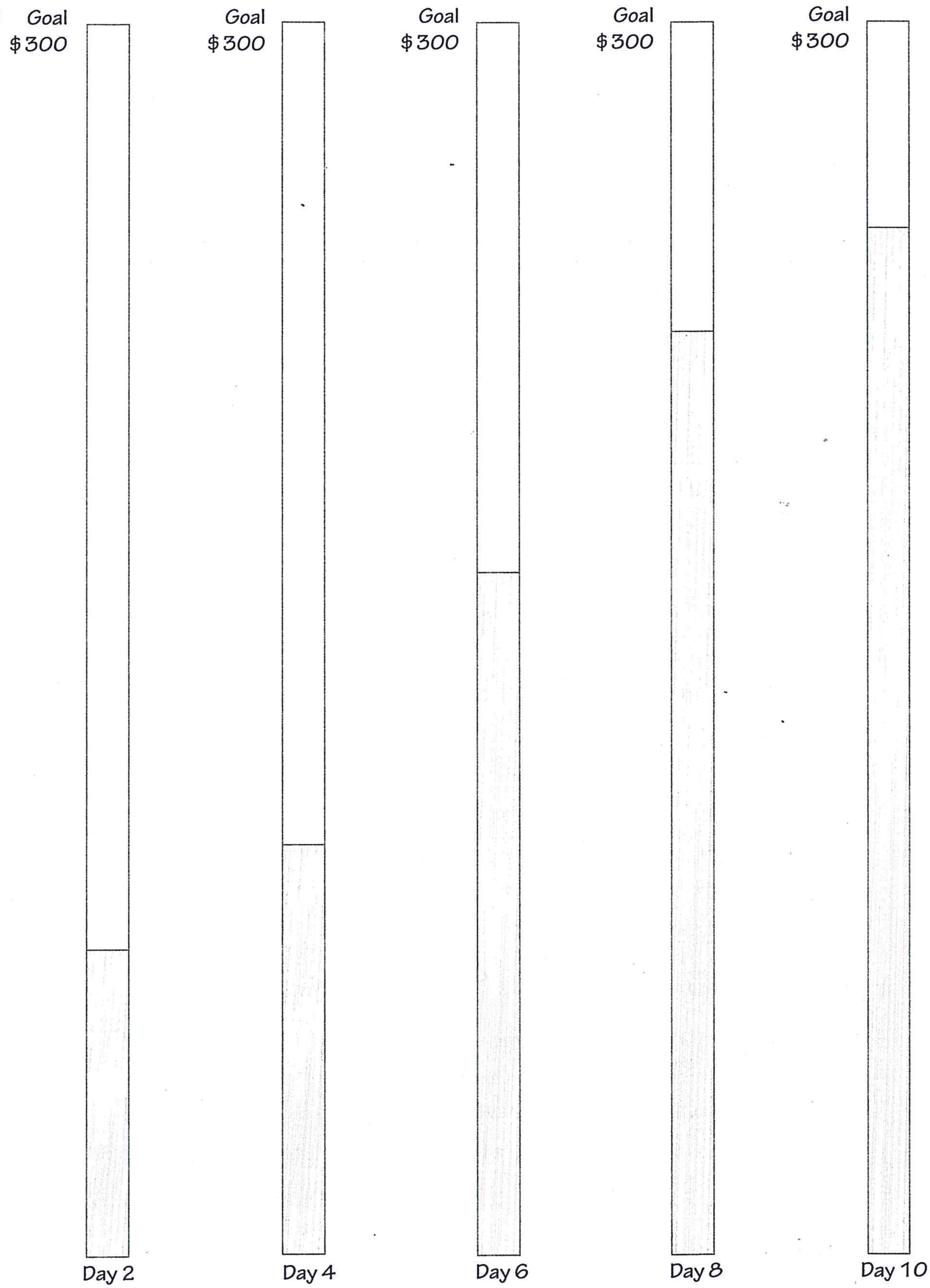
- Think about some ways in which working with fractions is like and not like working with ratios.

When you use fractions to compare a part to a whole, you often have more than one fraction name for the same quantity. For example, in Problem 1.3, you found that $\frac{1}{5} = \frac{2}{10}$.

In this next problem, you will compare the fundraising progress of a grade to its fundraising goal using fractions.

The thermometers on the next page show the progress of the sixth-grade poster sales after 2, 4, 6, 8, and 10 days. The principal needs to know what fraction of the goal the sixth grade has achieved after each day.

- How can you use your fraction strips to measure the sixth-grade's progress?


Goal
$300
Goal
$300
Goal
$300
Goal
$300
Goal
$300
Day 2
Day 4
Day 6
Day 8
Day 10

Problem 1.4

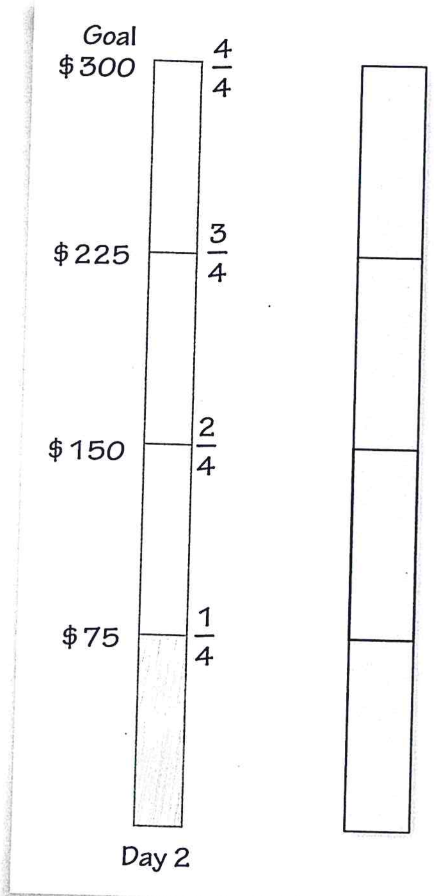
Examine the thermometers on the previous page and your fraction strips.

A How can you tell whether the sixth graders raised the same amount each day? Explain.

B What fraction of their goal did the sixth graders reach after Day 2? Day 4? Day 6? Day 8? Day 10?

C 1. Mary used her fourths strip to measure and label fractions and dollar amounts on the Day 2 thermometer at the right. Did she write the correct dollar amounts? How do you know?

2. Use your fraction strips to measure and label fraction and dollar amounts on copies of the remaining thermometers.



D 1. Jeri says that she can express the sixth-graders' progress on Day 2 in two ways using equivalent fractions: $\frac{1}{4}$ or $\frac{2}{8}$ of the goal. Find some other days for which you can write the sixth-graders' progress with two or more equivalent fractions.

2. Why do $\frac{1}{4}$ and $\frac{2}{8}$ both correctly describe the sixth-graders' progress on Day 2?

E At the end of Day 9, the sixth graders have raised $240.

1. What fraction of their goal have they reached?
2. Show how you would shade a blank thermometer for Day 9.

ACE Homework starts on page 27.

1.5 Comparing Fundraising Goals

Using Fractions and Ratios

In Problem 1.4, you used fractions to find parts of the sixth-graders' fundraising goal. Fraction strips and pictures such as fundraising thermometers are sometimes called **tape diagrams.** This is because a fraction strip is a long, skinny rectangle, like a long piece of tape.

In this Problem, you will use fractions to find parts of the other goals, and you will use ratios to compare the amounts raised by different grades.

A ratio comparison statement uses both numbers and words to show how two quantities are related. To write ratios, you can use the words *for every*, *to*, or a colon (:). For example, you may write these comparison statements.

For every \$60 dollars the sixth graders raise, the seventh graders raise \$90.

or

The ratio of the sixth-grade goal to the seventh-grade goal is 60 to 90.

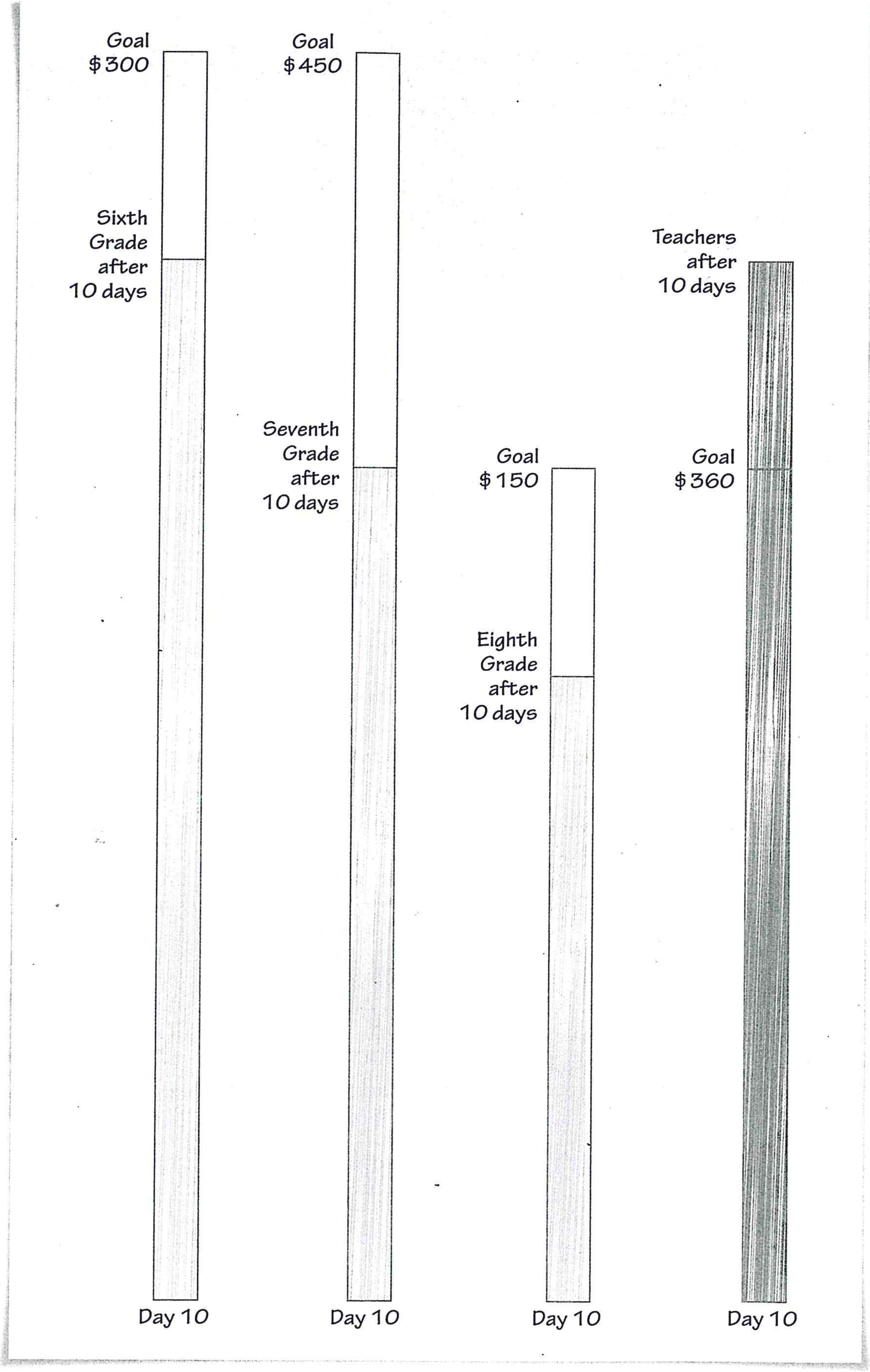
or

The ratio of the sixth-grade goal to the seventh-grade goal is 60 : 90.

You read the colon ":" using the word *to*. Both the word *to* and the colon are common in mathematics.

The fundraising thermometers on the next page show the goals and the progress of each grade and of the teachers after ten days.

- Which situations involve fractions? Ratios? How can you decide?


Goal
$300
Sixth
Grade
after
10 days
Day 10
Goal
$450
Seventh
Grade
after
10 days
Day 10
Goal
$150
Eighth
Grade
after
10 days
Day 10
Teachers
after
10 days
Goal
$360
Day 10

Problem 1.5

A **1.** What fraction of its goal did each grade reach by the end of Day 10 of the fundraiser?

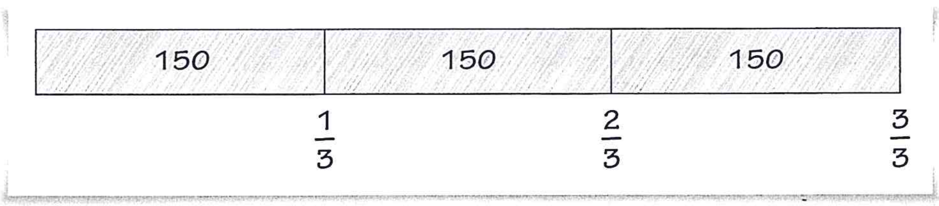
2. What fraction of their goal did the teachers reach by the end of Day 10 of the fundraiser?

3. How much money did each group raise?

B Margarita said: "I think the seventh graders raised \$300 by the end of Day 10 because I wrote several fractions that are equivalent to what I found with my fraction strips: $\frac{2}{3}$."

$$\frac{2}{3} = \frac{4}{6} = \frac{20}{30} = \frac{60}{90} = \frac{300}{450}$$

Margarita also drew this picture.



1. Explain how Margarita found these equivalent fractions. How does her picture relate to her method of finding equivalent fractions?

2. Use equivalent fractions to show how much money the sixth graders had raised by the end of Day 10.

3. Use equivalent fractions to show how much money the teachers had raised by the end of Day 10.

continued on the next page >

Problem 1.5 *continued*

C **1.** Brian wrote this comparison statement: The ratio of the amount of money raised by the sixth graders to the amount raised by the seventh graders is 250 : 300. Is this a correct statement? Explain.

2. Kate thought of $250 as 25 ten-dollar bills and $300 as 30 ten-dollar bills. She wrote the ratio, 25 : 30. Write a comparison statement using Kate's ratio.

3. Are Brian and Kate's two ratios equivalent? Explain.

4. What ratio would Kate write if she thought of $250 and $300 as numbers of fifty-dollar bills? Would thinking of twenty-dollar bills work? Explain.

5. Write two comparison statements, using equivalent ratios, for amounts of money raised by the sixth grade compared to the eighth grade in the fundraiser.

D On the last day of the fundraiser, the principal announces the results using both fractions and ratios. She has these two sticky notes on her desk.

$$\frac{250}{300} = \frac{5}{6}$$

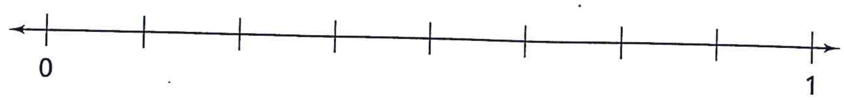
250 : 300
or
5 : 6

1. What do you think is the meaning of each note?

2. When are fractions useful? When are ratios useful?

ACE Homework starts on page 27.

6. Below is a number line labeled using an eighths strip. What other strips could label some of the marks on this number line?



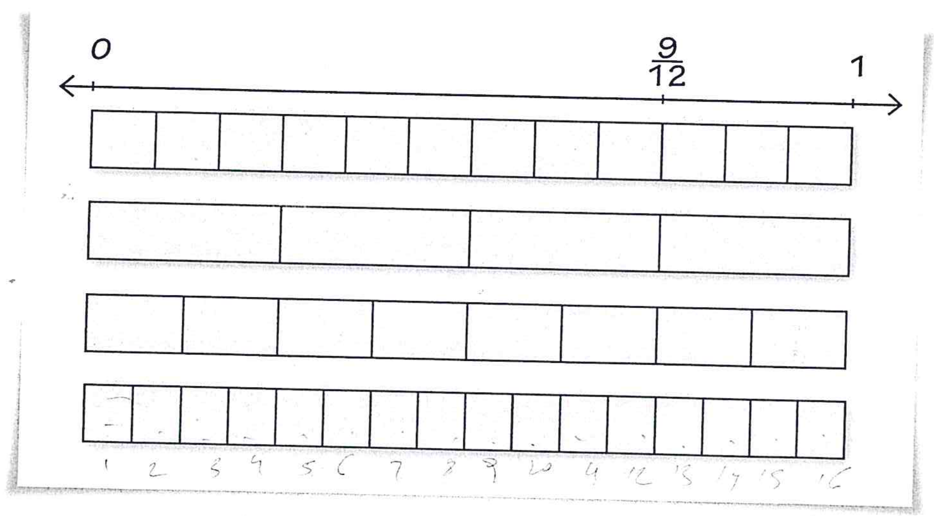
For Exercises 7–9, copy each number line. Make and use fraction strips or use some other method to estimate and name the point with a fraction.

7.


8.


9.


10. These students began to make a number line using different fraction strips as shown in the picture below. One student used the top fraction strip to mark $\frac{9}{12}$ on the number line.

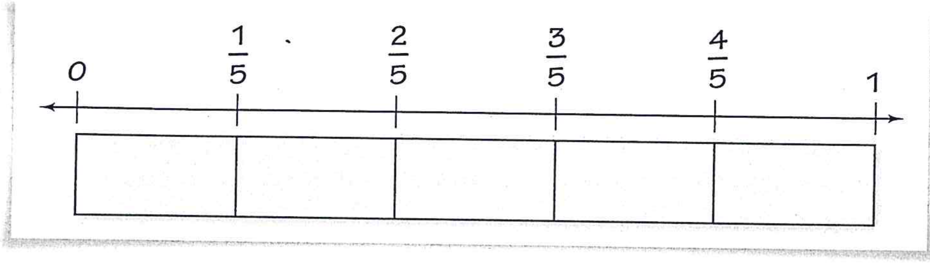


a. Name three other fractions shown here that are equivalent to $\frac{9}{12}$.

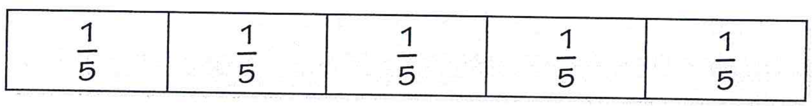
b. Name another fraction equivalent to $\frac{9}{12}$.

11. Erin used a fifths strip to mark and label $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$ on her number line, as shown below.

a. Why is no label needed for $\frac{5}{5}$?



b. Sally marked her fraction strip like this.



She says any two segments on her strip are the same as $\frac{2}{5}$. Do you agree with her? Explain how Sally's thinking is different from the way the number line is marked with $\frac{2}{5}$.

c. If you label marks for $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, $\frac{6}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, $\frac{9}{10}$, and $\frac{10}{10}$ on Erin's number line, which marks now have more than one label? Why is this?

d. If you were to extend your number line to reach from 0 to 2, there would be five fifths for every whole number length. What are some other "for every" statements you can make about a number line from 0 to 2?

For Exercises 12–15, decide whether the statement is correct or incorrect. Explain your reasoning in words or by drawing pictures.

12. $\frac{1}{3} = \frac{4}{12}$

13. $\frac{4}{6} = \frac{2}{3}$

14. $\frac{2}{5} = \frac{1}{3}$

15. $\frac{2}{5} = \frac{5}{10}$

For Exercises 16 and 17, use fraction strips to make marks on a number line to show that the two fractions are equivalent.

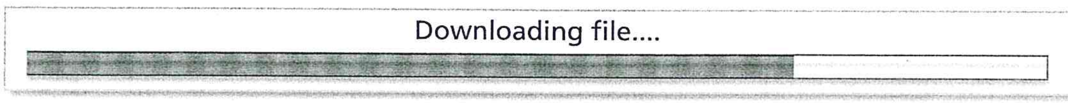
16. $\frac{2}{5}$ and $\frac{6}{15}$

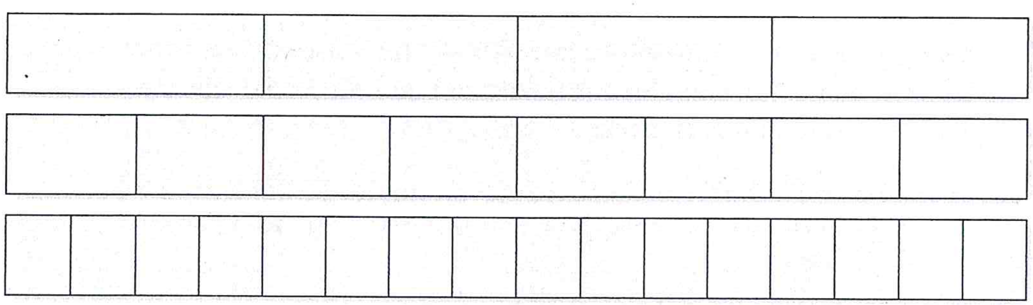
17. $\frac{1}{9}$ and $\frac{2}{18}$

18. Write an explanation to a friend telling how to find a fraction that is equivalent to $\frac{3}{5}$. You can use words and pictures to help explain.

19. When you save or download a file, load a program, or open a page on the Internet, a status bar is displayed on the computer screen to let you watch the progress.

a. Use the fraction strips shown to find three fractions that describe the status of the work in progress.



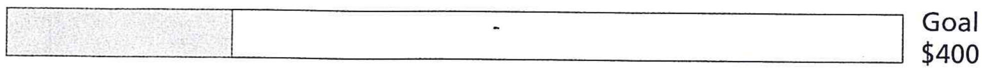


b. Suppose that you are downloading a movie with a file size of 2.8 GB (gigabyte). If the status bar above indicates how much of the movie has been downloaded, how many gigabytes have been downloaded so far?

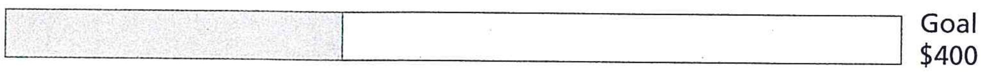
20. Use your fraction strips to locate and label these numbers on a number line: 0, $\frac{3}{4}$, and $\frac{7}{8}$. Then use your fraction strips to measure the distance between $\frac{3}{4}$ and $\frac{7}{8}$.

For Exercises 21 and 22, fold new fraction strips or use some other method to estimate the fraction of the fundraising thermometer that is shaded.

21.



22.



For Exercises 23–27, use this illustration of a drink dispenser. The gauge on the front of the dispenser shows how much of the liquid remains in the dispenser. The dispenser holds 120 cups.

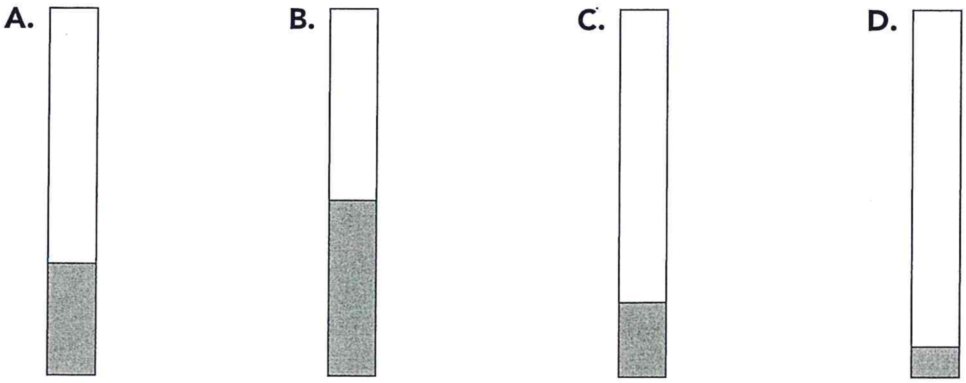
23. **a.** About what fraction of the dispenser is filled with liquid?

b. About how many cups of liquid are in the dispenser?

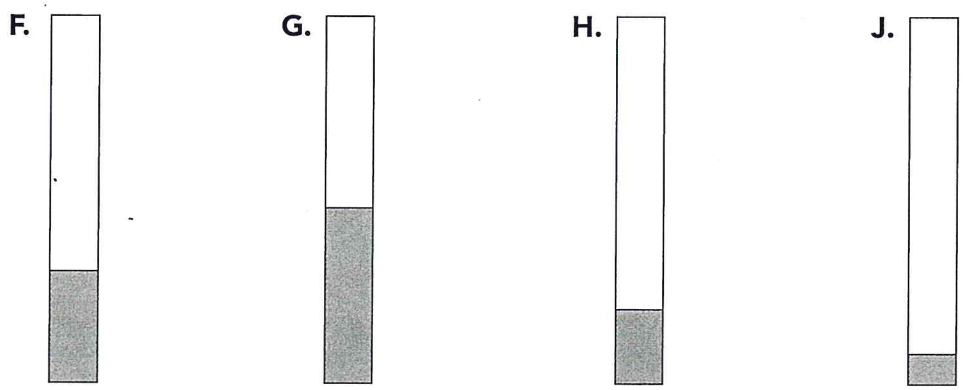
c. About what fraction of the dispenser is empty?

d. About how many more cups of liquid would it take to fill the dispenser?

24. Multiple Choice Which gauge shows about 37 out of 120 cups remaining?



25. Multiple Choice Which gauge shows about 10 out of 120 cups remaining?



26. In Exercises 24 and 25, about what fraction is shaded in each gauge you chose?

27. For parts (a)–(c), sketch the gauge and, for each dispenser, say whether it can be best described as *almost empty, about half full,* or *almost full.*

a. five sixths $\left(\frac{5}{6}\right)$ of a full dispenser

b. three twelfths $\left(\frac{3}{12}\right)$ of a full dispenser

c. five eighths $\left(\frac{5}{8}\right)$ of a full dispenser

28. If a class collects \$155 toward a fundraising goal of \$775, what fraction represents their progress toward their goal?

For Exercises 29–32, use the graphic below. Christopher downloads two different podcasts each day. Today, one file is loading more slowly than the other.

The Mathcast
45 MB of 60 MB - 22 seconds remaining

Fraction Podcast
20 MB of 30 MB - 1 minute remaining

29. What fraction of each file has downloaded so far?

30. Write a comparison statement for the sizes of the two files.

31. Write a comparison statement for the sizes of the downloaded parts of the two files.

32. How long will it take for each file to download, from beginning to end?

33. Dan, Karim, and Shawn are training for the school cross-country team. One day, they report the distances they ran as comparison statements.

a. Dan says he ran twice as far as Karim. Give three possibilities for the distances each could have run.

b. Karim says that the ratio of the distance he ran to the distance Shawn ran is 4 : 3. Give three possibilities for the distances each could have run.

c. Which boy ran the furthest?

34. Kate, Sue, and Lisa are on the school basketball team. After one game, they report their scoring as comparison statements.

a. Kate and Sue made the same number of successful shots as each other. Kate's successful shots were all 3-pointers. Sue's successful shots were all 2-pointers. Give three possibilities for the numbers of points each could have scored.

b. Lisa says that she made twice as many successful shots as Sue but scored the same number of points. How is this possible?

c. Which girl scored the most points?

d. Which girl made the most shots?

Connections

For Exercises 35–38, explain your answer to each question.

35. Is 450 divisible by 5, 9, and 10?

36. Is 12 a divisor of 48?

37. Is 4 a divisor of 150?

38. Is 3 a divisor of 51?

39. **Multiple Choice** Choose the number that is *not* a factor of 300.

A. 5

B. 6

C. 8

D. 20

40. **Multiple Choice** Choose the answer that shows all of the factors of 48.

F. 2, 4, 8, 24, and 48

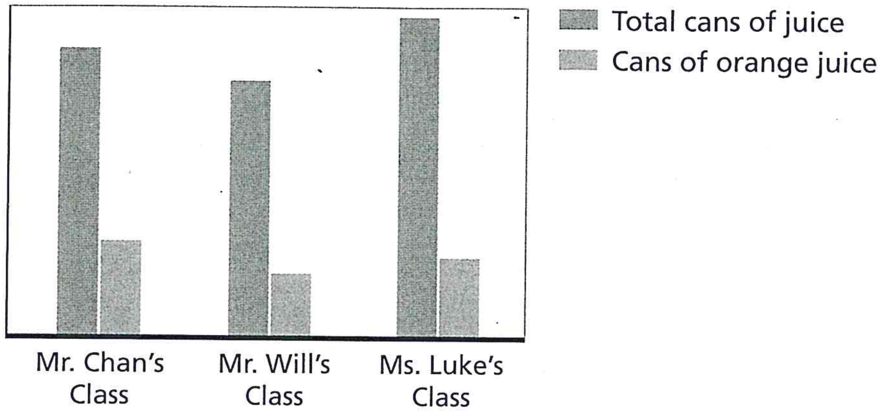
G. 1, 2, 3, 4, 5, 6, 8, and 12

H. 48, 96, and 144

J. 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48

For Exercises 41–43, use the bar graph below, which shows the number of cans of juice three sixth-grade classes drank.

Sixth-Grade Juice Consumption



41. In each class, what fraction of the cans were orange juice?

42. In which class would you say orange juice was most popular?

43. **a.** Students in Mr. Chan's class drank a total of ten cans of orange juice. About how many cans of orange juice did the students in each of the other two classes drink?

b. About how many total cans of juice did each of the three classes drink?

44. **a.** Miguel says that you can easily separate numbers divisible by 2 into two equal parts. Do you agree? Why or why not?

b. Manny says that if Miguel is correct, then you can easily separate numbers divisible by 3 into three equal parts. Do you agree? Why or why not?

c. Lupe says that if any number is divisible by n, you can easily separate it into n equal parts. Do you agree with her? Explain.

45. **a.** If you had a fraction strip folded into twelfths, what fractional lengths could you measure with the strip?

b. How is your answer in part (a) related to the factors of 12?

46. **a.** If you had a fraction strip folded into tenths, what fractional lengths could you measure with the strip?

b. How is your answer in part (a) related to the factors of 10?

47. Ricky found a beetle that is one fourth $\left(\frac{1}{4}\right)$ the length of the fraction strips used in Problem 1.3.

a. How many beetle bodies, placed end to end, would have a total length equal to the length of a fraction strip?

b. How many beetle bodies, placed end to end, would have a total length equal to three fraction strips?

c. Ricky drew 13 paper beetle bodies, end to end, each the same length as the one he found. How many fraction strips long is Ricky's line of beetle bodies?

48. Rachel looked at the two ratios 25 : 30 and 250 : 300. In each ratio she noticed that the first and second numbers have a common factor.

a. What are some common factors of 25 and 30?

b. What are some common factors of 250 and 300?

c. Rachel says that the two numbers in a ratio will always have a common factor. Is she correct?

49. Abby looked at the same ratios (25 : 30 and 250 : 300). In these two equivalent ratios, she noticed that the first numbers have a common factor and the second numbers have a different common factor.

a. What are some common factors of 25 and 250?

b. What are some common factors of 30 and 300?

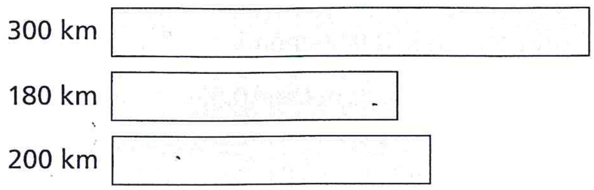
c. Abby says that the first numbers in two equivalent ratios will always have a common factor. Is she correct?

For Exercises 50 and 51, write a fraction to describe how much pencil is left, compared to a new pencil. Measure from the left edge of the eraser to the point of the pencil.

50. 

51. 

52. These bars represent trips that Ms. Axler took in her job this week.



a. Copy each bar and shade in the distance Ms. Axler traveled after going one third of the total distance for each trip.

b. How many kilometers had Ms. Axler traveled when she was at the one-third point in each trip? Explain your reasoning.

53. Brett and Jim sign up to run in the Memorial Day race in their town. There are two different events at this race, a 5K (5 kilometers) and a 10K (10 kilometers). Brett signed up for the 5K and Jim signed up for the 10K.

a. Make fraction strips where each kilometer run is partitioned on equal length fraction strips for both Brett and Jim.

b. Use thermometers to indicate when both Brett and Jim have finished $\frac{3}{5}$ of their races. How many kilometers has each person run at this point?

c. Use the thermometers to indicate when both Brett and Jim are finished with four kilometers of their races. What fraction represents the amount of their respective races they have finished?

d. Write a "for every" claim that relates the distances Brett and Jim have run to their distance goals.

54. A sprinter finished a 100-meter race in a time of 12.63 seconds.

a. If the sprinter were able to keep the same rate of speed, how long would it take him to complete the 10,000-meter race?

b. A long-distance runner won first place in the 10,000-meter race with a time of 37 minutes, 30 seconds. What is the time difference between the long-distance runner's actual time and the sprinter's hypothetical time from part (a)?

55. **Multiple Choice** Find the least common multiple of the following numbers: 3, 4, 5, 6, 10, and 15.

A. 1 **B.** 15

C. 60 **D.** 54,000

56. Use what you found in Exercise 55. Write the following fractions in equivalent form, all with the same denominator.

$$\frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{6} \quad \frac{1}{10} \quad \frac{1}{15}$$

For Exercises 57–60, find the greatest common factor of each pair of numbers.

57. 12 and 48

58. 6 and 9

59. 24 and 72

60. 18 and 45

For Exercises 61–64, use your answers from Exercises 57–60 to write a fraction equivalent to each fraction given.

61. $\frac{12}{48}$

62. $\frac{6}{9}$

63. $\frac{24}{72}$

64. $\frac{18}{45}$

Extensions

For Exercises 65–67, write a numerator for each fraction to make the fraction close to, but not equal to, $\frac{1}{2}$. Then, write another numerator to make each fraction close to, but greater than, 1.

65. $\frac{\square}{22}$ **66.** $\frac{\square}{43}$ **67.** $\frac{\square}{17}$

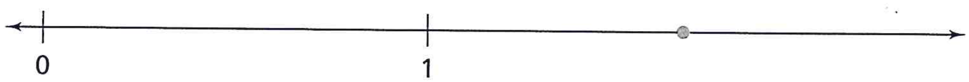
For Exercises 68–70, write a denominator to make each fraction close to, but not equal to, $\frac{1}{2}$. Then, write another denominator to make each fraction close to, but greater than, 1.

68. $\frac{22}{\square}$ **69.** $\frac{43}{\square}$ **70.** $\frac{17}{\square}$

For Exercises 71–74, copy the number line. Use your knowledge of fractions to estimate and name the point with a fraction.

71.

72.



73.

74.

Mathematical Reflections 1

In your work in this Investigation, you wrote comparison statements using fractions and ratios. You also used fraction strips to make a number line and compare fractions. These questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. a. **Write** three comparison statements about the same situation, one using difference, one using a fraction, and one using a ratio.

   b. **Explain** what you think a ratio is.

2. a. **What** does it mean for two fractions to be equivalent? For two ratios to be equivalent?

   b. **What** are some useful ways of finding equivalent fractions and equivalent ratios?

Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Sophie described her thoughts in the following way:

We used fraction strips and number lines as a tool in Problem 1.3 to compare fractions and to find groups of fractions related to each other.

When we lined up the fraction strips, we noticed that equivalent fractions occurred when the fold marks lined up with each other and that these were all names for the same point on the line.

Common Core Standards for Mathematical Practice

MP5 Use appropriate tools strategically

- What other Mathematical Practices can you identify in Sophie's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Investigation 2

Connecting Ratios and Rates

In Investigation 1, you used fraction strips as a tool to determine the fraction of each fundraising goal reached and locate points and distances on a number line. You also used ratios to compare quantities and checked to see if they were equivalent. In this Investigation you will continue to explore ratios and ways to write equivalent ratios.

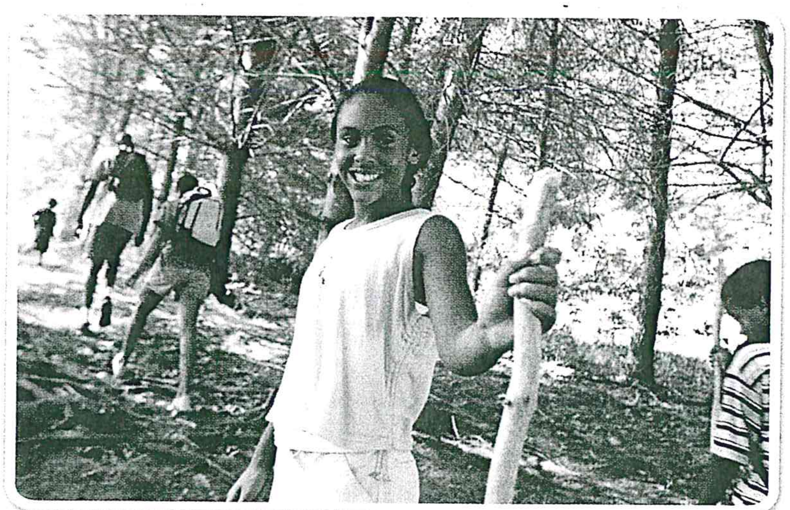
The ratio statements in Investigation 1 were written as "for every" or "to" statements. Ratios can be written in many different ways.

Suppose the cost for ten students to go on a field trip is $120. You can write ratios to show how the quantities are related.

10 students *for every* $120

10 students *to every* $120

10 students : $120



Common Core State Standards

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.A.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables . . .

6.RP.A.3b Solve unit rate problems including those involving unit pricing and constant speed.

Also 6.RP.A.1, 6.RP.A.2, 6.NS.B.4

Ratio statements can also be written as "per" statements. For example, "It costs \$120 per 10 students to go on the trip." An equivalent comparison statement is "the cost per student to go on a field trip is \$12." Now you can say

\$12 *for every* 1 student

\$12 *for each* student

\$12 *per* student

This particular comparison, cost per one student, is called a unit rate. A **unit rate** is a comparison in which one of the numbers being compared is 1 unit.

- If the cost of food is \$250 for 50 students, what is the cost per student?

To answer this question, you find the unit rate.

2.1 Equal Shares
Introducing Unit Rates

Often we share food so that each person gets the same amount. This may mean that food is cut into smaller pieces. Think about how to share a chewy fruit worm that is already marked in equal-sized pieces.

The chewy fruit worm below shows four equal segments.

How can you share this 4-segment chewy fruit worm equally among four people?

How many segments of the worm does each person get?

OR

How can you share this 4-segment chewy fruit worm equally among three people?

How many segments of the worm does each person get?

Problem 2.1

In Questions A and B, find the fraction of a chewy fruit worm each person gets.

A 1. Show two ways that four people can share a 6-segment chewy fruit worm. In each case, how many segments does each person get?

2. Show two ways that six people can share an 8-segment chewy fruit worm. In each case, how many segments does each person get?

B 1. Show how 12 people can share an 8-segment chewy fruit worm. How many segments are there for every person?

2. Show how five people can share a 3-segment chewy fruit worm. How much is this per person?

C Jena wants to share a 6-segment chewy fruit worm. The tape diagram below shows the marks she made on the worm so she can share it equally among the members in her CMP group.

1. How many people are in her group?
2. Is there more than one possible answer to part (1)? Explain.
3. What is the number of segments per person?
4. Write a fraction to show the part of the chewy fruit worm each person gets.

D Would you rather be one of four people sharing a 6-segment chewy fruit worm or one of eight people sharing a 12-segment chewy fruit worm? Explain.

E Look back at your work on this Problem. Describe how you found or used unit rates.

ACE Homework starts on page 50.

2.2 Unequal Shares

Using Ratios and Fractions

Sometimes there are reasons to share quantities *unequally*. Suppose your older brother paid more than half the cost of a video game. You might think it is fair for him to spend more time playing the game. At a party, you might agree that your friend should take the bigger piece of chocolate cake because your friend likes chocolate more than you do.

Two sisters, Crystal and Alexa, are going to a strange birthday party. Instead of birthday cake, pairs of party guests are each served a large chewy fruit worm to share according to their ages. Since the sisters are not the same age, they do not share their fruit worm equally.

Crystal is 12 years old and Alexa is 6 years old. Their chewy fruit worm has 18 segments. According to their ages, Crystal gets 12 segments and Alexa gets 6 segments. The ratio of the girls' shares of the worm, 12 to 6, is equivalent to the ratio of their ages, 12 to 6.

- According to the rule, how would the girls share a 9-segment chewy fruit worm?

Since Crystal's age is two times Alexa's age, Crystal gets twice as many segments as Alexa. The ratio of Crystal's segments to Alexa's segments is 12 to 6 or 2 to 1.

- The ratio 2 to 1 is a unit rate. What do the numbers 2 and 1 mean for the sisters?

In this Problem you will explore situations that involve fractions and ratios.

Problem 2.2

A Draw some chewy fruit worms with different numbers of segments that Crystal and Alexa can share without having to make new cuts.

B 1. Jared is 10 years old. His brother Peter is 15 years old. What are some chewy fruit worms they can share without having to make new cuts?

2. For each worm you described in part (1), write a ratio comparing the number of segments Jared gets to the number of segments Peter gets.

3. Are the ratios you wrote in part (2) equivalent to each other? Explain.

4. How would you write a unit rate to compare how many segments Jared and Peter get?

C 1. Caleb and Isaiah are brothers. They share a 14-segment chewy fruit worm according to their age. How old could they be?

2. Caleb gets 8 out of the 14 segments of the chewy fruit worm, so he gets $\frac{8}{14}$ and Isaiah gets $\frac{6}{14}$ of the worm.

   a. From Question A, what fractions of the chewy fruit worm do Crystal and Alexa each get at the birthday party?

   b. From Question B, what fractions of the chewy fruit worm do Jared and Peter each get at the birthday party?

   c. How does the ratio of segments that Caleb and Isaiah get relate to the fractions of the chewy fruit worm that they each get?

ACE Homework starts on page 50.

2.3 Making Comparisons With Rate Tables

When comparing how to share chewy fruit worms, Crystal recorded how many segments she and her sister would get for different sizes of chewy fruit worms. Crystal thought she could use what she knew about equivalence to make a table showing the amounts.

Comparing Segments

Segments for Alexa	6	3	1	2	$\frac{1}{2}$	10
Segments for Crystal	12	6	2	4	1	20

The table shows that for every segment given to Alexa, Crystal gets two segments. This is Alexa's unit rate. The table also shows that for every $\frac{1}{2}$ segment Alexa is given, Crystal gets one segment. This is Crystal's unit rate.

Crystal sees an ad for chewy fruit worms. She decides she wants the student council to include chewy fruit worms in the fundraising sale.

You can use the information in the advertisement to compute the price for any number of worms you want to buy. One way to figure out the price of a single item from a quantity price is use the information to build a **rate table** of equivalent ratios.

The rate table in Question A shows the price for different numbers of chewy fruit worms. The cost of 30 chewy fruit worms is $3.

Problem 2.3

A **1.** Crystal wants to calculate costs quickly for many different numbers of chewy fruit worms. Copy and complete the rate table below with prices for each of the numbers of chewy fruit worms.

Chewy Fruit Worm Pricing

Number of Worms	1	5	10	15	30	90	150	180
Reduced Price	■	■	■	■	$3	■	■	■

2. How much do 3 chewy fruit worms cost? 300 chewy fruit worms?

3. How many chewy fruit worms can you buy for $50? For $10?

4. What is the unit price of one chewy fruit worm? What is the unit rate?

B The student council also decides to sell popcorn to raise money. One ounce of popcorn (unpopped) kernels yields 4 cups of popcorn. One serving is a bag of popcorn that holds 2 cups of popcorn.

1. Use a rate table to find the number of ounces of popcorn kernels needed to determine the cups of popcorn.

Cups of Popcorn From Ounces of Kernels

Number of Cups of Popcorn	4	■	■	■	■	■	■	■	■	■	■	■
Number of Ounces of Popcorn Kernels	1	2	3	4	5	6	7	8	9	10	11	12

2. How many cups of popcorn can you make from 12 ounces of popcorn kernels? From 30 ounces of popcorn kernels?

3. How many ounces of popcorn kernels are needed to make 40 cups of popcorn? To make 100 cups of popcorn?

4. How many ounces of kernels are needed to make 100 servings?

5. How many ounces of kernels are needed to make 1 cup?

C **1.** How do rate tables help you answer Question A and Question B?

2. How do unit rates help you answer Question A and Question B?

ACE Homework starts on page 50.

Applications | Connections | Extensions

Applications

1. Show two ways three people can share a 5-segment chewy fruit worm.

2. Show two ways five people can share a 3-segment chewy fruit worm.

3. Sharon is ready to share the 4-segment chewy fruit worm shown below. She has already made the marks she needs so that she can share it equally among the members of her group.

   a. Give two different numbers of people that could be in Sharon's group.

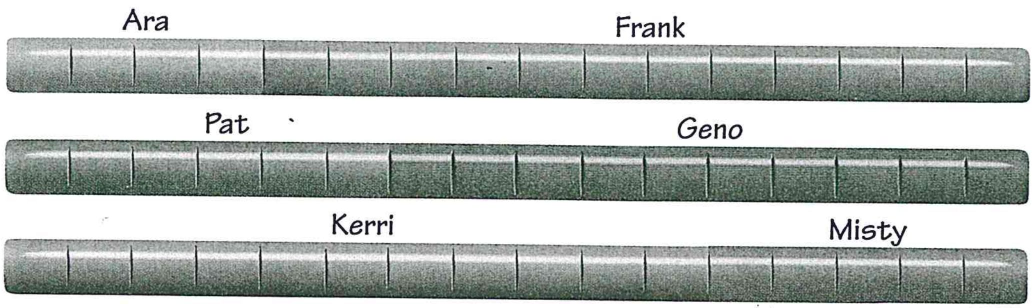
   b. For each answer you gave in part (a), write a ratio comparing the number of people sharing a chewy fruit worm to the number of segments they are sharing. How would you rewrite this as a unit rate?

4. Cheryl, Rita, and four of their friends go to a movie and share a 48-ounce bag of popcorn equally and three 48-inch licorice laces equally. Write a ratio comparing the number of ounces of popcorn to the number of friends. Then, write a unit rate comparing the length of licorice lace for each person.

5. The Lappans buy three large sandwiches to serve at a picnic. Nine people come to the picnic. Show three different ways to cut the sandwiches so that each person gets an equal share.

6. Three neighbors are sharing a rectangular strip of land for a garden. They divide the land into 24 equal-sized pieces. They each get the same amount of land. Write a ratio comparing the number of pieces of land to the number of people. Write the answer in more than one way.

7. For each chewy fruit worm below write the possible ages of the two people sharing the worm by age.



Use this information for Exercises 8–10. At the birthday party in Problem 2.2, the children run relay races. The distance each team member runs depends on the ratio of their ages. For example, a boy who is twice as old as a girl runs twice as far.

8. Crystal is 12 years old and Alexa is 6 years old. If Crystal runs 100 yards, how far does Alexa run? How far do they run altogether?

9. Jared is 10 years old and Peter is 15 years old. Together, they run 150 yards. How far does each brother run?

10. Wynne and Emmett are brother and sister. Wynne runs 180 yards. Emmett runs 120 yards. How old could each of them be?

Use this information for Exercises 11–14. Parents are older than their children. The ratio of a parent's age to a child's age changes as the parent and child get older.

11. Can a parent ever be exactly twice as old as his or her child? Explain.

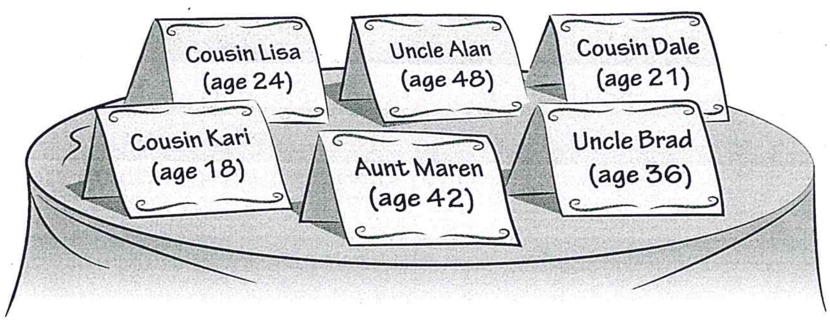
12. Can a parent ever be exactly three times as old as his or her child? Explain.

13. Can the ratio of a parent's age to his or her child's age ever be exactly 3 : 2? Explain.

14. Can the ratio of a parent's age to his or her child's age ever be exactly 10 : 9? Explain.

15. Crystal and Alexa convince the older members of their family to break up the chewy fruit worms using age ratios. They want to know which family members have the same age ratio as Crystal and Alexa.

a. Use the ages of their family members to find pairs that have the same age ratio as Crystal (age 12) and Alexa (age 6).



b. What do all the ratios that you wrote in part (a) have in common?

For Exercises 16–18, copy and complete the table comparing the chewy fruit worm segments each family member received. State both unit rates in each comparison.

16.

Segments for Alan	48	12	■	1	■	7
Segments for Lisa	24	■	8	■	1	■

17.

Segments for Lisa	24	12	■	1	■	■
Segments for Alexa	6	■	2	■	1	$1\frac{1}{2}$

18.

Segments for Alan	48	24	■	1	■	■
Segments for Alexa	6	■	2	■	1	$1\frac{1}{2}$

For Exercises 19–22, use the family members from Exercise 15, including Crystal and Alexa. Determine which two people have each age ratio.

19. The unit rate is 2 : 1.

20. The unit rate is 4 : 1.

21. The ratio of segments (ages) is 3 : 4.

22. The ratio of segments (ages) is 3 : 2.

For Exercises 23 and 24, Rosco is planning meals for his family. He uses the vertical rate tables.

23. **a.** Complete the rate table for the macaroni and cheese ingredients.

Macaroni and Cheese

Ounces of Macaroni	Cups of Cheese
8	1
■	2
■	3
■	4
■	5
■	6

b. How many ounces of macaroni would you need for 7 cups of cheese?

c. How many cups of cheese would you need for 88 ounces of macaroni?

24. a. Complete the rate table for the spaghetti ingredients.

Spaghetti and Sauce

Ounces of Spaghetti	Ounces of Tomatoes
12	16
6	8
3	
2	
1	

b. What is the unit rate comparing the number of ounces of tomatoes to 1 ounce of spaghetti?

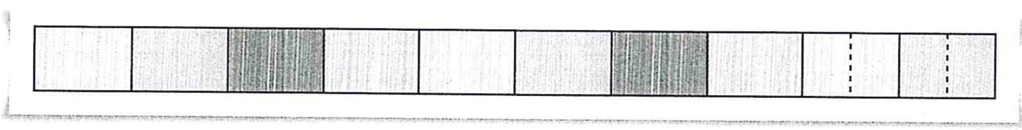
c. What is the unit rate comparing 1 ounce of tomatoes to the number of ounces of spaghetti?

Connections

25. Ursula, Ubaldo, Ulysses, and Dora were trying to come up with different ways to divide a 10-segment chewy fruit worm among the four of them. Which of these strategies would result in sharing equally?

- Ursula's Strategy:

  Give everyone two segments, and then divide the remaining two segments into four equal pieces with each person getting another half of a segment.



- Ubaldo's Strategy:

  Give each person one segment, then if there's at least four segments left, give each person another segment. Repeat this process until there are less than four segments, then cut the leftover pieces into four equal parts and give each person a part.

- Ulysses' Strategy:

  Give each person two segments, and then use a spinner to pick the winner of the extra two segments.

- Dora's Strategy:

  Forget about the segments. Just cut the worm in half, and then cut each half in half again.

26. If you were going to make segment marks on a chewy fruit worm without any marks, what would be the advantage or disadvantage of using a prime number of segments?

27. A typical container of orange juice concentrate holds 12 fluid ounces (fl oz). The standard recipe is "Mix one can of concentrate with three cans of cold water."

a. What is the ratio of concentrate to water?

b. How large of a container will you need to hold the juice?

c. Olivia has a one-gallon container to fill with orange juice. She uses the standard recipe. How much concentrate does she need? (One gallon is 128 fl oz.)

28. A typical container of lemonade concentrate holds 12 fl oz. The standard recipe is "Mix one can of concentrate with $4\frac{1}{3}$ cans of cold water."

a. What is the ratio of concentrate to water?

b. How large of a container will you need to hold the lemonade?

c. Olivia has a one-gallon container to fill with lemonade. She uses the standard recipe. How much concentrate does she need? (One gallon is 128 fl oz.)

29. Langhus Convenience Store sells multiple sizes of chewy fruit worms. Betsy, Emily, and John are trying to decide which of the deals would give them the most chewy fruit worms for the price.

a. Which argument do you think is the best? Explain.

- Betsy: The small size is the best deal because you get the most amount of worms, 10 more than the medium size, and 18 more than the large size.
- John: The large size is the best deal because you have to pay the least amount of money overall.
- Emily: I used the least common multiple of 4, 8, and 12, which is 24. For $24, I could buy 60 large worms, 54 medium worms, and 56 small worms. The large size is the best deal.

b. How could Betsy, John, and Emily use unit rates to find the best deal?

30. As Johann is working on unit rates in Exercises 16–24, he notices something interesting and says to his teacher, "Whenever you compare two quantities and you write both unit rates, at least one of them will have a fraction in it." Is Johann correct? Explain why you agree or disagree with him.

Extensions

For Exercises 31–33, consider the conjectures Jena made while working on Problem 2.1. Which conjectures do you think are true? Explain.

31. If the number of people is greater than the number of segments, each person will get less than one segment.

32. There are at least two ways to divide any chewy fruit worm so that everyone will get the same amount.

33. If the ratio of people to segments is 1 : 2, then each person will get $\frac{1}{2}$ of a segment.

34. Harold is eight years older than Maynard. On Harold's sixteenth birthday, he notices something interesting about their age ratios. He says, "When I was nine, the ratio of my age to Harold's was 9 : 1. A year later the ratio was 5 : 1. That's when I was ten and Maynard was two. Now on my sixteenth birthday, I'm twice as old as Maynard, which means the ratio of our ages is 2 : 1." Will Harold and Maynard ever have an age ratio 1 : 1? Explain.

35. A women's 4-by-100 meter medley relay team finished in second place. In the relay, each member swims 100 meters using a different stroke. The ages of the team members are 21, 22, 25, and 41.

The age difference between the oldest and youngest swimmer on this team was 20 years!

Suppose they had broken up the distance of 400 meters by age as in Problem 2.2. How far would each person swim in the relay?

36. Mariette, Melissa, and Michelle were given this follow-up question by Mr. Mirasola to Problem 2.3, "If you had \$3.55, how many large chewy fruit worms could you buy?"

- Mariette said that she could buy $35\frac{1}{2}$.
- Melissa said that she could buy only 35.
- Michelle said that she could buy only 30.

Mr. Mirasola said, "You are all correct depending on how you think of the ad." How is it possible that they could all be correct?

37. On a recent trip to Canada, Tomas learned that there was an "exchange rate" between U.S. dollars and Canadian dollars. When he exchanged his U.S. dollars, he did not get the same number of Canadian dollars back. Tomas hopes to visit many different countries one day, so he does some research and finds a Web site with some basic money conversions on it.

a. Find the unit rate for each country below.

Currency Exchange Rates

$20 US ≈ 19 Australian Dollars	$1 US ≈ ■ AUD	$■ US ≈ 1 AUD
$5 US ≈ 4 Euros	$1 US ≈ ■ Euros	$■ US ≈ 1 Euro
$50 US ≈ 49 Swiss Francs	$1 US ≈ ■ SF	$■ US ≈ 1 SF
$3 US ≈ 2 Pounds (UK)	$1 US ≈ ■ Pounds	$■ US ≈ 1 Pound
$4 US ≈ 5 Singapore Dollars	$1 US ≈ ■ SGD	$■ US ≈ 1 SGD

Note: Exchange rates often change from day to day; there are Web sites that have the most up-to-date exchange rates.

b. How can you use this information to convert euros to Australian dollars or Swiss francs to Singapore dollars? Explain.

Mathematical Reflections 2

In this Investigation, you used ratios to share equally and unequally according to certain rules. You used rate tables and unit rates to solve problems. These questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary in your notebook.

1. a. **How** can you determine a unit rate for a situation?

   b. **Describe** some ways that unit rates are useful.

2. a. **What** strategies do you use to make a rate table?

   b. **Describe** some ways that rate tables are useful.

3. **How** are your strategies for writing equivalent ratios the same as or different from writing equivalent fractions?

Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of the Problems. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Jayden described his thoughts in the following way:

> We used rate tables to find the prices for different amounts of chewy fruit worms in Problem 2.3.
>
> In the rate table, we noticed a repeated pattern such as "for every 5 worms we need to pay $.50." Some of us expressed this pattern in the amount of a unit rate: the money per each worm or number of worms per $1.
>
> In figuring out how much we need to pay for 300 worms, we used our rate table and noticed that there is a $3 increase for every 30 worms.
>
> **Common Core Standards for Mathematical Practice**
> **MP7** Look for and make use of structure

- What other Mathematical Practices can you identify in Jayden's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.